# ECE 476 – Power System Analysis Fall 2017
## Homework 6

**Reading**: Chapter 6 of textbook
**In-class quiz:** Tuesday October 17, 2017

**Problem 1.** Compute the elements of the **third** row of $Y_{bus}$ for the power system in Example 6.9 of textbook.

**Solution.** We find each entry in the third row of the admittance matrix as follows:

$$\bar{Y}_{31} = \bar{Y}_{32} = \bar{Y}_{35} = 0.$$

$$\bar{Y}_{34} = \frac{-1}{R_{34} + jX_{34}} = \frac{-1}{0.00075 + j0.01} = -7.458 + j99.44 \text{ p.u.}$$

$$\bar{Y}_{33} = \frac{1}{R_{34} + jX_{34}} + j\frac{B'_{34}}{2} = \frac{1}{0.00075 + j0.01} + j\frac{0}{2} = 7.458 - j99.44 \text{ p.u.}$$

So the third row of the admittance matrix is

$$\bar{Y}_3 = \begin{bmatrix} 0 & 0 & 7.458 - j99.44 & -7.458 + j99.44 & 0 \end{bmatrix}.$$

**Problem 2.** Given the impedance diagram of a simple system as shown in Figure 1, draw the admittance diagram for the system and develop the 4 x 4 bus admittance matrix $Y_{bus}$ by inspection.

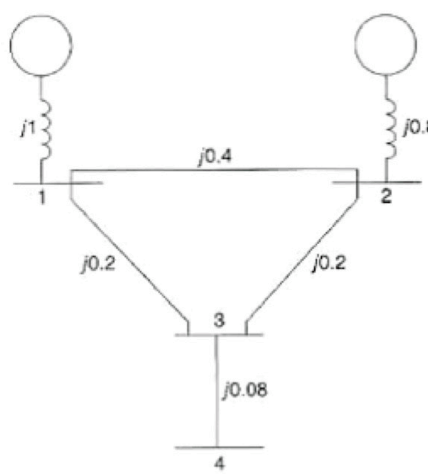


Figure 1: System diagram for problem 2.

**Solution.** The admittance diagram is shown in Figure 2.

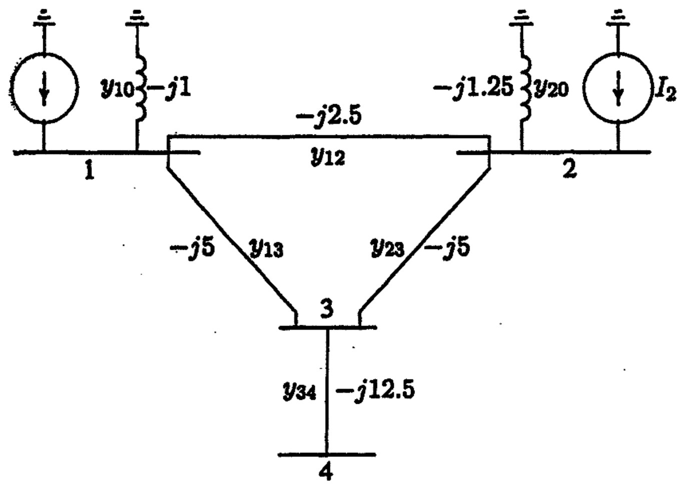


Figure 2: Admittance diagram for Problem 2

The structure of the admittance matrix is given as

$$\bar{Y}_{\text{bus}} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \bar{Y}_{13} & \bar{Y}_{14} \\ \bar{Y}_{21} & \bar{Y}_{22} & \bar{Y}_{23} & \bar{Y}_{24} \\ \bar{Y}_{31} & \bar{Y}_{32} & \bar{Y}_{33} & \bar{Y}_{34} \\ \bar{Y}_{41} & \bar{Y}_{42} & \bar{Y}_{43} & \bar{Y}_{44} \end{bmatrix},$$

where the components are found as follows

$$\bar{Y}_{11} = \bar{y}_{10} + \bar{y}_{12} + \bar{y}_{13} \qquad \bar{Y}_{22} = \bar{y}_{20} + \bar{y}_{12} + \bar{y}_{23}$$
$$\bar{Y}_{23} = \bar{y}_{13} + \bar{y}_{23} + \bar{y}_{34} \qquad \bar{Y}_{44} = \bar{y}_{34}$$
$$\bar{Y}_{12} = \bar{Y}_{21} = -\bar{y}_{12} \qquad \bar{Y}_{13} = \bar{Y}_{31} = -\bar{y}_{13}$$
$$\bar{Y}_{23} = \bar{Y}_{32} = -\bar{y}_{23} \qquad \bar{Y}_{34} = \bar{Y}_{43} = -\bar{y}_{34}.$$

Plugging in the values from the admittance diagram, the admittance matrix becomes

$$\bar{Y}_{\text{bus}} = j \begin{bmatrix} -8.5 & 2.5 & 5.0 & 0 \\ 2.5 & -8.75 & 5.0 & 0 \\ 5.0 & 5.0 & -22.5 & 12.5 \\ 0 & 0 & 12.5 & -12.5 \end{bmatrix} \text{S}.$$

**Problem 3.** A load $L$ consuming 1 p.u. of active power and 0.5 p.u. of reactive power is connected to a generator $G1$ through a short transmission line with $Z' = 0.02 + j0.06$ p.u. Also, there is a capacitor connected to the load bus with admittance $Y_{cap} = j0.25$ p.u. The generator voltage is voltage $V_{G1} = 1\angle 0°$.

**Solution**

a) The one line diagram is shown in Figure 3.

b) The admittance matrix can be found by inspection as

$$\overline{Y} = \begin{bmatrix} 5 - j15 & -5 + j15 \\ -5 + j15 & 5 - j14.75 \end{bmatrix}.$$

c) The power flow equations can now be written for each bus.
Bus 1:

$$P_1 = V_1^2 G_{11} + V_1 V_2 [G_{12}\cos(\theta_1 - \theta_2) + B_{12}\sin(\theta_1 - \theta_2)] = 5 + V_2[-5\cos(\theta_2) - 15\sin(\theta_2)]$$
$$Q_1 = -V_1^2 B_{11} + V_1 V_2 [G_{12}\sin(\theta_1 - \theta_2) - B_{12}\cos(\theta_1 - \theta_2)] = 15 + V_2[5\sin(\theta_2) - 15\cos(\theta_2)]$$

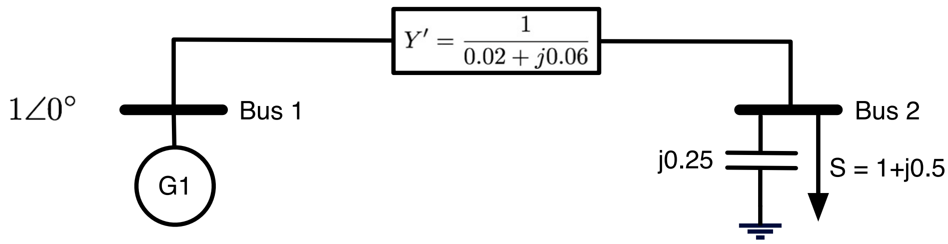


Figure 3: One Line Diagram

Bus 2:

$$P_2 = V_2^2 G_{22} + V_2 V_1[G_{21}\cos(\theta_2 - \theta_1) + B_{21}\sin(\theta_2 - \theta_1)] = 5V_2^2 + V_2[-5\cos(\theta_2) + 15\sin(\theta_2)] = -1$$
$$Q_2 = -V_2^2 B_{22} + V_2 V_1[G_{21}\sin(\theta_2 - \theta_1) - B_{21}\cos(\theta_2 - \theta_1)] = 14.75V_2^2 + V_2[-5\sin(\theta_2) - 15\cos(\theta_2)] = -0.5$$

**Problem 4.** Solve the following equation by the Newton-Raphson method:

$$2x_1^2 + x_2^2 - 8 = 0$$
$$x_1^2 - x_2^2 + x_1 x_2 - 4 = 0$$

Start with an initial guess of $x_1 = 1$ and $x_2 = 1$.

**Solution.** Define $\boldsymbol{x} = [x_1, x_2]^\top$, and

$$\boldsymbol{f} = \begin{bmatrix} 2x_1^2 + x_2^2 - 8 \\ x_1^2 - x_2^2 + x_1 x_2 - 4 = 0 \end{bmatrix}.$$

Then,

$$\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} = \begin{bmatrix} 4x_1 & 2x_2 \\ 2x_1 + x_2 & -2x_2 + x_1 \end{bmatrix}.$$

The update rule using Newton-Raphson method is:

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} - \left(\frac{\partial \boldsymbol{f}(\boldsymbol{x}^{(k)})}{\partial \boldsymbol{x}}\right)^{-1} \boldsymbol{f}(\boldsymbol{x}^{(k)})$$

After 4 iterations, you will get the solution $\boldsymbol{x} = [1.8091, 1.2060]^\top$.

**Problem 5.** Assume a $1+j0.5$ per unit load at bus 2 is being supplied by a generator at bus 1 through a transmission line with series impedance of $0.05 + j0.1$ per unit. Assuming bus 1 is the swing bus with a fixed per unit voltage of $1.0\angle 0^\circ$, use Newton-Raphson method to calculate the voltage at bus 2 after three iterations.

**Solution.** Similar to Problem 3, the admittance matrix can be found by inspection as

$$\overline{Y} = \begin{bmatrix} 4 - j8 & -4 + j8 \\ -4 + j8 & 4 - j8 \end{bmatrix}.$$

The power flow equations at bus 2 are

$$P_2 = V_2^2 G_{22} + V_2 V_1[G_{21}\cos(\theta_2 - \theta_1) + B_{21}\sin(\theta_2 - \theta_1)] = 4V_2^2 + V_2[-4\cos(\theta_2) + 8\sin(\theta_2)] = -1$$
$$Q_2 = -V_2^2 B_{22} + V_2 V_1[G_{21}\sin(\theta_2 - \theta_1) - B_{21}\cos(\theta_2 - \theta_1)] = 8V_2^2 + V_2[-4\sin(\theta_2) - 8\cos(\theta_2)] = -0.5$$

Define $\boldsymbol{x} = [\theta_2, V_2]^\top$, and

$$\boldsymbol{f} = \begin{bmatrix} 4V_2^2 - 4V_2\cos\theta_2 + 8V_2\sin\theta_2 + 1 \\ 8V_2^2 - 4V_2\sin\theta_2 - 8V_2\cos\theta_2 + 0.5 \end{bmatrix}.$$

Then,

$$\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} = \begin{bmatrix} 4V_2\sin\theta_2 + 8V_2\cos\theta_2 & 8V_2 - 4\cos\theta_2 + 8\sin\theta_2 \\ -4V_2\cos\theta_2 + 8V_2\sin\theta_2 & 16V_2 - 4\sin\theta_2 - 8\cos\theta_2 \end{bmatrix}.$$

The update rule using Newton-Raphson method is:

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} - \left(\frac{\partial \boldsymbol{f}(\boldsymbol{x}^{(k)})}{\partial \boldsymbol{x}}\right)^{-1} \boldsymbol{f}(\boldsymbol{x}^{(k)})$$

The iteration process is shown below:

$$\boldsymbol{x}^{(0)} = [0, 1]^\top$$
$$\boldsymbol{f}(\boldsymbol{x}^{(0)}) = [1.0, 0.5]^\top$$
$$\frac{\partial \boldsymbol{f}(\boldsymbol{x}^{(0)})}{\partial \boldsymbol{x}} = \begin{bmatrix} 8 & 4 \\ 0 & 8 \end{bmatrix}$$

$$\boldsymbol{x}^{(1)} = [-0.0938, 0.9375]^\top$$
$$\boldsymbol{f}(\boldsymbol{x}^{(1)}) = [0.0800, 0.4152]^\top$$
$$\frac{\partial \boldsymbol{f}(\boldsymbol{x}^{(1)})}{\partial \boldsymbol{x}} = \begin{bmatrix} 7.1160 & 2.7687 \\ -0.3510 & 7.4096 \end{bmatrix}$$

$$\boldsymbol{x}^{(2)} = [-0.0834, 0.8820]^\top$$
$$\boldsymbol{f}(\boldsymbol{x}^{(2)}) = [0.0800, 0.4152]^\top$$
$$\frac{\partial \boldsymbol{f}(\boldsymbol{x}^{(2)})}{\partial \boldsymbol{x}} = \begin{bmatrix} 7.1160 & 2.7687 \\ -0.3510 & 7.4096 \end{bmatrix}$$

$$\boldsymbol{x}^{(3)} = [-0.0834, 0.8820]^\top$$
$$\boldsymbol{f}(\boldsymbol{x}^{(3)}) = [0.0082, -0.0146]^\top$$